MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

DEM5046 – ALGEBRA & TRIGONOMETRY

(Diploma in Electronic Engineering)

11 MARCH 2020
2:30 p.m. - 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages (excluding the cover page and appendices) with 5 Questions only.

2. Answer ALL **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Write your answers in the answer booklet provided. *All relevant working steps must be shown clearly.*

QUESTION 1

(a) Simplify and write the following expression so that all exponents are positive.

$$\frac{x^{-3}y^4}{x^5y^{12}}$$

[2 marks]

(b) Multiply the following polynomial using the FOIL (First, Outer, Inner, Last) method. Express your answer as a single polynomial in standard form

$$(-2x + 10y)(-2x + 8y)$$

[2 marks]

(c) Factor the following polynomial completely by grouping:

$$3x^3 + 4x^2 - 6x - 8$$

[3 marks]

(d) Simplify by combining the following like radicals:

$$\sqrt{98x^3y} + 3x\sqrt{32xy}$$

[3 marks]

(e) Simplify the following complex rational expression using the Least Common Multiple (LCM) Method:

$$\frac{\frac{1}{2} + \frac{6}{x}}{\frac{x+3}{4}} \quad x \neq -3, 0$$

[5 marks]

(f) Factor and simplify the following expression:

$$\frac{4}{3}x^{\frac{1}{3}}(2x + 1) + 2x^{\frac{4}{3}}$$

[5 marks]

Continued ...

QUESTION 2

(a) Solve the linear equation using a calculator. Round the answer to two decimal places.

$$8.81x + \frac{49}{12.231} = 23.03$$

[2 marks]

(b) Donald and Boris are both travelling to their university for their morning lecture class. It was found that Donald requires 30 minutes for his trip. On the other hand, Boris needs 45 minutes because he lives 20 kilometers (km) farther away as compared to Donald. In addition, Boris travels 5 km per hour faster than Donald. Calculate each of their average travelling rates (in km per hour).

[4 marks]

(c) Find the real solutions, if any, of the equation. Write your answer in solution set format:

$$9 + \frac{3}{x} - \frac{2}{x^2}$$

[5 marks]

(d) Solve the following combined inequality:

$$-2 < 1 - \frac{x}{2} < 5$$

[3 marks]

(e) Solve the following inequality using the sign diagram method:

$$\frac{1}{x+2} \geq \frac{3}{x+1}$$

[6 marks]

Continued ...

QUESTION 3

(a) For a given line segment with endpoints (8, -4) and (-6, 1), find the following:

(i) Distance between the two endpoints. Truncate your answer to 1 decimal point. [2 marks]

(ii) Coordinates of the midpoint M for the line segment. [2 marks]

(b) Consider a line containing the points (2, 3) and (-4, 5):

(i) Find the two equations for that line. [5 marks]

(ii) Sketch the line. [2 marks]

(c) For a given circle with radius 4 and center (-3, 2),

(i) Write the standard form of an equation for the circle. [2 marks]

(ii) Write the general form of the equation for the circle. [2 marks]

(iii) Find the intercepts, if any, of its graph. Round your answer to 2 decimal points. [5 marks]

QUESTION 4

(a) You have been given a project to design a water sprinkler which is required to cover a field of $50m^2$ that is in the shape of a sector of a circle with radius 10m. Based on your calculations for the area of a sector of a circle theory, calculate the following:

(i) the required angle (in radians) that the water sprinkler needs to rotate. [3 marks]

(ii) convert your answer to degrees. Round your answer to 1 decimal point. [2 marks]

(b) Given that $\sin\theta = \frac{1}{3}$ and θ is an acute angle, use the Pythagorean Identity to find the remaining values for the remaining five trigonometric functions of θ. [5 marks]

(c) Sketch the graph $y = -3\cos\pi x$ over one period using transformations. Explain each step clearly. [6 marks]

(d) If $\tan\theta = \frac{4}{3}$, $\pi < \theta < \frac{3}{2}\pi$, find the exact value of:

(i) $\cos 2\theta$ [2 marks]

(ii) $\sin\frac{\theta}{2}$ [2 marks]

Continued...

QUESTION 5

(a) Solve the equation $x^4 + 3x^2 - 4 = 0$ in the complex number system. [3 marks]

(b) Find the complex fourth roots of $4 - 4\sqrt{3}i$ using De Moivre's Theorem. Express the answers in exponential form. [8 marks]

(c) Use Cramer's Rule, if applicable, to solve the following system:

$$\begin{cases} x + 4y - 3z = -8 \\ 3x - y + 3z = 12 \\ x + y + 6z = 1 \end{cases}$$

[9 marks]

End of paper.

APPENDIX

A Standard Form of Circle Equation

$$(x-h)^2+(y-k)^2=r^2$$

B Fundamental Identities

- Even & Odd Properties:
$$\sin(-\theta)=-\sin\theta \qquad \csc(-\theta)=-\csc\theta$$
$$\cos(-\theta)=\cos\theta \qquad \sec(-\theta)=\sec\theta$$
$$\tan(-\theta)=-\tan\theta \qquad \cot(-\theta)=-\cot\theta$$

- Reciprocal identities:
$$\csc\theta=\frac{1}{\sin\theta},\ \sec\theta=\frac{1}{\cos\theta},\ \cot\theta=\frac{1}{\tan\theta}$$

- Pythagorean identities :
$$\sin^2\theta+\cos^2\theta=1,\quad \tan^2\theta+1=\sec^2\theta,\quad 1+\cot^2\theta=\csc^2\theta$$

- Sum/Difference Formula for Cosine:
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

- Sum/Difference Formula for Sine:
$$\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$$

- Sum/Difference Formula for Tangent:
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

- Double Angle Formula:
$$\cos 2A=\cos^2 A-\sin^2 A$$
$$=1-2\sin^2 A$$
$$=2\cos^2 A-1$$
$$\sin 2A=2\sin A\cos A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

- Half Angle Formula:
$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

C Relations between Polar and Rectangular Coordinates

$x = r\cos\theta$; $y = r\sin\theta$; $r = \sqrt{x^2 + y^2}$; $\tan\theta = \frac{y}{x}$

D Products and Quotients in Polar Form

- $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]; \quad z_2 \neq 0$

E DeMoivre's Theorem

- Powers of Complex Numbers on Polar Form:
  $z^n = r^n(\cos n\theta + i\sin n\theta)$
- Roots of Complex Numbers in Polar Form:
  $z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right]$
  $k = 0, 1, 2, \ldots, n-1$

F Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x\tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\csc x] = -\csc x\cot x$

G The Chain Rule

If $y = f[g(x)]$ and $u = g(x)$, then $y = f(u)$:

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ or $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$

H Derivative of the Exponential & logarithmic Function

$$\frac{d}{dx}[e^x] = e^x; \qquad \frac{d}{dx}[e^u] = e^u.\frac{du}{dx}; \qquad \frac{d}{dx}\left(a^x\right) = a^x \ln a$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0 \quad ; \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}.\frac{du}{dx};$$

$$\frac{d}{dx}\log_a x = \frac{1}{\ln a}\cdot\frac{1}{x} = \frac{1}{x\ln a}$$

I Derivative of Inverse Trigonometry Function

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\csc^{-1} x\right] = -\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = -\frac{1}{1+x^2}$$

J Derivative of Hyperbolic Function

$$\frac{d}{dx}[\sinh x] = \cosh x \qquad \frac{d}{dx}[\cosh x] = \sinh x \qquad \frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$$

$$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}[\coth x] = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}[\operatorname{csch} x] = -\operatorname{csch} x \coth x$$

K The Second Derivative Test

Suppose f'' is continuous near c.

If $f'(c) = 0$ and $f''(c) > 0$, then f has local minimum at c.

If $f'(c) = 0$ and $f''(c) < 0$, then f has local maximum at c.